

one sample t test example problem with solution

one sample t test example problem with solution is a fundamental concept in statistics used to determine whether the mean of a single sample significantly differs from a known or hypothesized population mean. This statistical test is widely applied in various fields such as psychology, medicine, business, and social sciences to make informed decisions based on sample data. Understanding the formulation, calculation, and interpretation of the one sample t test is essential for accurate data analysis. This article provides a detailed explanation of the one sample t test, including assumptions, step-by-step procedures, and a comprehensive example problem with a solution. Additionally, the article covers how to interpret the results and common pitfalls to avoid. By the end, readers will gain a robust understanding of how to apply the one sample t test effectively in practical scenarios.

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Understanding the One Sample t Test

The one sample t test is a statistical method used to compare the mean of a single sample to a known or hypothesized population mean. It helps to determine if the observed sample mean significantly deviates from the expected population mean, considering sample variability and size. This test is especially useful when the population standard deviation is unknown and the sample size is relatively small, making the standard normal distribution inappropriate to use.

The test relies on the t-distribution, which adjusts for the uncertainty in estimating the population standard deviation from a sample. The one sample t test produces a t-statistic, which quantifies the difference between the sample mean and the population mean relative to the sample's standard error. This t-statistic is then compared to critical values from the t-distribution to decide whether to reject the null hypothesis.

Assumptions of the One Sample t Test

Before applying the one sample t test, several key assumptions must be met to ensure valid results. These assumptions are critical for the accuracy and reliability of the test outcomes.

Assumption 1: Normality

The data in the sample should be approximately normally distributed. This assumption is especially important for small sample sizes (less than 30). For larger samples, the Central Limit Theorem helps mitigate concerns about normality.

Assumption 2: Independence

The observations in the sample must be independent of each other, meaning the value of one observation does not influence another.

Assumption 3: Scale of Measurement

The variable under study should be measured at the interval or ratio scale to calculate meaningful means and standard deviations.

Step-by-Step Procedure for the One Sample t Test

Conducting a one sample t test involves several systematic steps that ensure clarity and accuracy in hypothesis testing. Below is the typical procedure followed in applying this test.

1. **State the hypotheses:** Formulate the null hypothesis (H_0) as the population mean equals a specific value, and the alternative hypothesis (H_a) as the population mean differing from that value.
2. **Choose the significance level (α):** Commonly set at 0.05, this defines the threshold for rejecting the null hypothesis.
3. **Calculate the sample mean ($\bar{x}$) and sample standard deviation (s):** Summarize the sample data through these statistics.
4. **Compute the t-statistic:** Use the formula $t = (\bar{x} - \mu_0) / (s / \sqrt{n})$, where μ_0 is the hypothesized population mean, s is the sample standard deviation, and n is the sample size.
5. **Determine degrees of freedom (df):** Calculated as $n - 1$.
6. **Find the critical t-value:** Based on the significance level and degrees of freedom, use t-distribution tables or software.

7. **Make a decision:** Compare the calculated t-statistic with the critical t-value to decide whether to reject or fail to reject the null hypothesis.

One Sample t Test Example Problem with Solution

To illustrate the application of the one sample t test, consider the following example problem with solution. This example demonstrates all steps and calculations clearly.

Problem Statement

A nutritionist claims that the average sodium content in a brand of canned soup is 800 mg per serving. To verify this claim, a sample of 15 cans is tested, and the sodium content values (in mg) are recorded. The sample mean is found to be 830 mg, with a standard deviation of 40 mg. Using a significance level of 0.05, test whether the average sodium content differs from the claimed 800 mg.

Step 1: State the Hypotheses

- Null hypothesis (H_0): $\mu = 800$ mg (the mean sodium content is 800 mg)
- Alternative hypothesis (H_a): $\mu \neq 800$ mg (the mean sodium content is not 800 mg)

Step 2: Significance Level

The significance level is $\alpha = 0.05$.

Step 3: Calculate Sample Statistics

Given: sample mean ($\bar{x}$) = 830 mg, sample standard deviation (s) = 40 mg, sample size (n) = 15.

Step 4: Calculate the t-Statistic

The formula for the t-statistic is:

$$t = (\bar{x} - \mu_0) / (s / \sqrt{n})$$

Substituting the values:

$$t = (830 - 800) / (40 / \sqrt{15}) = 30 / (40 / 3.873) = 30 / 10.33 \approx 2.90$$

Step 5: Degrees of Freedom

Degrees of freedom, $df = n - 1 = 15 - 1 = 14$.

Step 6: Critical t-Value

For a two-tailed test at $\alpha = 0.05$ with $df = 14$, the critical t-values are approximately ± 2.145 .

Step 7: Decision

The calculated t-statistic (2.90) is greater than the critical t-value (2.145). Therefore, the null hypothesis is rejected.

Interpretation

There is sufficient evidence at the 0.05 significance level to conclude that the average sodium content in the canned soup differs from the claimed 800 mg.

Interpreting the Results of the One Sample t Test

Interpreting the results of a one sample t test involves understanding what rejecting or failing to reject the null hypothesis means in the context of the data and research question.

Rejecting the Null Hypothesis

If the calculated t-statistic exceeds the critical t-value, the null hypothesis is rejected. This implies that the sample provides strong evidence that the population mean is significantly different from the hypothesized mean.

Failing to Reject the Null Hypothesis

If the calculated t-statistic does not exceed the critical value, there is insufficient evidence to reject the null hypothesis. This suggests that the sample data does not show a significant difference from the hypothesized population mean.

p-Value Approach

Alternatively, the p-value can be used to interpret results. If the p-value is less than the significance level (α), reject the null hypothesis; otherwise, fail to reject it. This approach provides a probability measure of observing the sample data under the assumption that the null hypothesis is

true.

Common Mistakes and Considerations

When performing a one sample t test, several common mistakes and important considerations should be kept in mind to ensure the validity of conclusions.

- **Ignoring Assumptions:** Applying the test without checking normality or independence can lead to invalid results.
- **Using the Wrong Test:** If the population standard deviation is known, a z-test may be more appropriate than a t-test.
- **Misinterpreting Results:** Failing to distinguish between statistical significance and practical significance can mislead conclusions.
- **Small Sample Sizes:** Very small samples may not provide reliable estimates; consider non-parametric alternatives if assumptions are violated.
- **Two-Tailed vs. One-Tailed Tests:** Choose the correct alternative hypothesis type based on the research question to avoid incorrect conclusions.

Frequently Asked Questions

What is a one sample t test and when is it used?

A one sample t test is a statistical method used to determine whether the mean of a single sample differs significantly from a known or hypothesized population mean. It is used when the population standard deviation is unknown and the sample size is small.

Can you provide a simple example problem of a one sample t test with solution?

Example: A teacher claims that the average test score of her class is 75. A random sample of 10 students has an average score of 72 with a sample standard deviation of 5. Test at the 0.05 significance level whether the average score differs from 75. Solution: Null hypothesis $H_0: \mu = 75$; Alternative hypothesis $H_1: \mu \neq 75$. Calculate $t = (72 - 75) / (5 / \sqrt{10}) = -1.897$. Degrees of freedom = 9. Critical t value at 0.05 significance (two-tailed) $\approx \pm 2.262$. Since $-1.897 > -2.262$, we fail to reject H_0 . There is not enough evidence to say the average score differs from 75.

How do you calculate the test statistic in a one sample t test?

The test statistic t is calculated using the formula $t = (\bar{x} - \mu) / (s / \sqrt{n})$, where $\bar{x}$ is the sample mean, μ is the population mean under the null hypothesis, s is the sample standard deviation, and n is the sample size.

What assumptions must be met to perform a valid one sample t test?

The assumptions are: 1) The sample data are independent and randomly selected. 2) The data are approximately normally distributed, especially important for small sample sizes. 3) The population standard deviation is unknown.

How do you interpret the results of a one sample t test example problem?

If the calculated t statistic falls into the critical region (beyond the critical t value), or if the p -value is less than the chosen significance level (e.g., 0.05), you reject the null hypothesis, indicating a significant difference between the sample mean and the population mean. Otherwise, you fail to reject the null hypothesis.

Can a one sample t test be used for large sample sizes?

Yes, a one sample t test can be used for large sample sizes, but for very large samples, the t distribution approaches the normal distribution, and a z test may be more appropriate. However, the one sample t test remains valid regardless of sample size if the population standard deviation is unknown.

Additional Resources

1. *Understanding the One-Sample T-Test: A Practical Approach*

This book offers a clear introduction to the one-sample t -test, explaining when and how to use it effectively. It includes step-by-step example problems and detailed solutions that help readers grasp the concept thoroughly. Perfect for beginners, it focuses on real-world applications in various fields such as psychology, business, and medicine.

2. *Statistics Made Simple: One-Sample T-Test Explained*

Designed for students and professionals alike, this book breaks down the one-sample t -test into easy-to-understand segments. It provides numerous example problems, complete with solutions and interpretations, to ensure readers can apply the test confidently. The emphasis on simplicity makes complex statistical ideas accessible.

3. Applied Statistics with One-Sample T-Tests: Theory and Practice

This comprehensive guide balances theoretical background with practical application of the one-sample t-test. It features example problems drawn from real datasets and walks readers through the solution process in detail. The book is ideal for those looking to deepen their understanding of hypothesis testing.

4. One-Sample T-Test Workbook: Examples and Solutions

A hands-on workbook filled with a variety of example problems involving one-sample t-tests, accompanied by fully worked-out solutions. It allows readers to practice at their own pace and reinforce their understanding of concepts such as assumptions, calculation steps, and interpretation of results. Great for self-study and classroom use.

5. Introductory Statistics: One-Sample T-Test with Step-by-Step Solutions

This introductory text demystifies the one-sample t-test through clear explanations and a wealth of example problems. Each problem is followed by a detailed solution, highlighting common pitfalls and tips for accurate analysis. It serves as an excellent resource for students new to statistics.

6. Practical Guide to Hypothesis Testing: One-Sample T-Test Examples

Focusing on hypothesis testing, this guide presents the one-sample t-test as a key example. The book includes numerous problem scenarios, from simple to complex, complete with thorough solutions and interpretations. Readers gain a practical understanding of how to test means against known values.

7. Statistics in Action: Mastering the One-Sample T-Test

This book aims to build mastery in performing one-sample t-tests by providing detailed examples and problem-solving strategies. It covers the underlying assumptions, formula derivations, and stepwise calculation processes. The included examples come with comprehensive solutions to reinforce learning.

8. Data Analysis Essentials: One-Sample T-Test Explained with Examples

Focused on data analysis, this book explains the one-sample t-test within the broader context of statistical inference. It provides example problems with solutions that demonstrate how to analyze and interpret data effectively. The text is practical and designed to enhance analytical skills.

9. Hands-On Statistics: One-Sample T-Test Problems and Solutions

This resource is packed with practical problems involving the one-sample t-test, each accompanied by a detailed solution. It encourages active learning through problem-solving and offers insights into common mistakes and best practices. Ideal for learners who prefer a hands-on approach to statistics.

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